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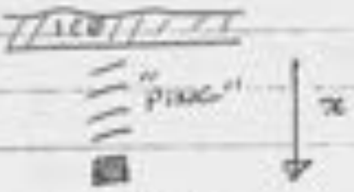
Mobile Robotics Q Sheet 2

1) BOOK WORK

a) $\vec{x} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$ ← position, ← velocity



$p(x)$ density

ASSUME CONSTANT VELOCITY MODEL – NOISE IS IN ACCELERATION

$$\vec{x}(k) = F\,\vec{x}(k-1) + B\,\vec{q}(k)$$

$F = \underbrace{\begin{bmatrix} 1 & \Delta T \\ 0 & 1 \end{bmatrix}}$ OPEN LOOP EXPECTED EVOLUTION OF PLAN

$B = \underbrace{\begin{bmatrix} \frac{\Delta T^2}{2} & \Delta T \end{bmatrix}^T}$ PROPAGATION OF NOISE INTO STATE

q: SCALAR ACCEL NOISE $\sim N(0, q^2)$ OR $N(0, Q)$, Q A 1X1 MATRIX

N.B IN COVARIANCE EQUATION WE WOULD HAVE A BQB^T TERM WHICH EVALUATES TO $\begin{bmatrix} \frac{\Delta T^4}{4} & \frac{\Delta T^3}{2} \\ \frac{\Delta T^3}{2} & \Delta T^2 \end{bmatrix} q^2$

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$$\vec{z}(k) = H\,\vec{x}(k) + \vec{w}(k)$$

EXPLAINS OBS IN TERM OF STATE

NOISE IN SONAR READING (TIME)

TWO WAY TRIP

WE ONLY OBSERVE POSITION

$$H = \frac{2}{c}\begin{bmatrix}1 & 0\end{bmatrix}$$

SPEED OF SOUND

z IS A SCALAR (TIME)

$\vec{w}$ IS A SCALAR $\sim N(0, R)$ ← A 1x1 MATRIX

d) THIS IS WHY WE HAVE COMPUTERS!

IMPORTANT POINT IS THAT BOTH STATE COMPONENTS x & $\dot{x}$ ARE MENTION IN MODELS. ALTHOUGH WE ONLY OBSERVE POSITION (VIA T.O.F) THE PLANT MODEL PROPAGATES CORRELATIONS BETWEEN x & $\dot{x}$. SO WHEN A CHANGE IN x IS MEASURED IT ALSO CAUSES A CHANGE IN ESTIMATED $\dot{x}$.

$$P(0|0) = \begin{bmatrix}\infty & 0\\ 0 & \infty\end{bmatrix} \qquad x(0|0) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$P(1|0) = F\,P(0|0)\,F^T + BQB^T \qquad x(1|0) = F\,x(0|0)$$

$$= \begin{bmatrix}\infty & \infty\\ \infty & \infty\end{bmatrix} \qquad = \begin{bmatrix}0\\0\end{bmatrix}$$

1st PREDICTION

THEN TAKE MEASUREMENT AND UPDATE

SCALAR IN THIS PROBLEM

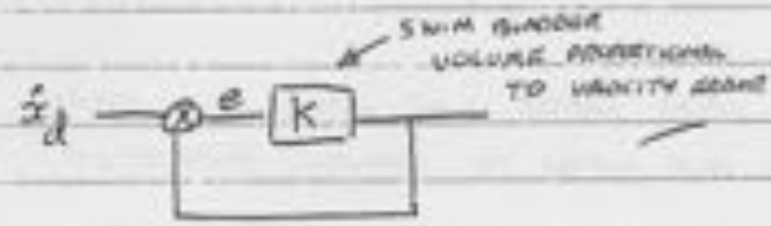
$$\cancel{x(1|1)}\; x(1|1) = x(1|0) + W\big(z(k) - H\,x(1|0)\big)$$

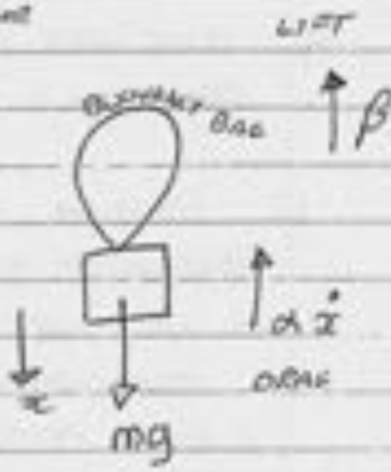
$$W = P H^T S^{-1} = \begin{bmatrix}\infty & \infty\\ \infty & \infty\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix}^T \times \text{SCALAR}^{-1}$$

$$= \begin{bmatrix}\infty\\ \infty\end{bmatrix}$$ ← ALL COMPONENTS OF $W\nu(k)$ ARE NON-ZERO

ALL OF $\vec{x}$ IS CORRECTED INCLUDING VELOCITY!

2 c)



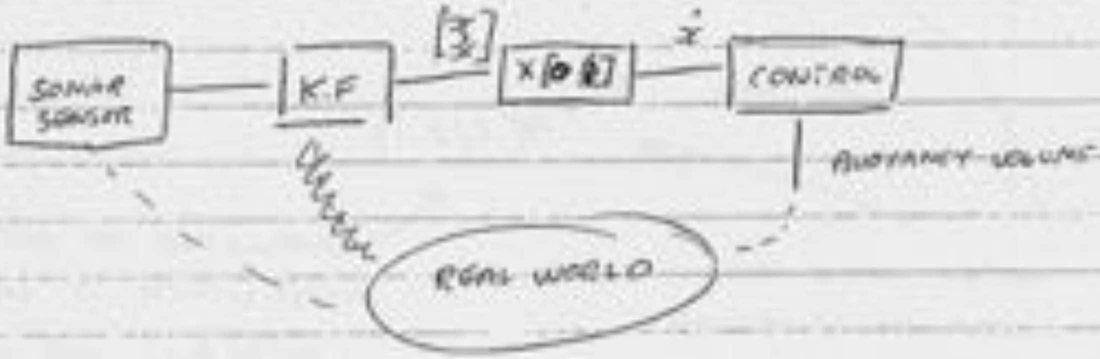


$$m\ddot{x} = -\alpha\dot{x} - \beta + mg$$

$$m\ddot{x} + \alpha\dot{x} = mg - \beta$$

WANT THIS TO BE ZERO

$\rightarrow \dot{x}$ prop β



3a) BOOK WORK FROM NOTES – NO NEED TO LEARN EQUATIONS – BUT NEED TO LEARN ANNOTATIONS!

b) JACOBIANS ARE LINEARISATION OF PLANT (F) AND OBSERVATION MODEL WHEN THEY ARE NON-LINEAR – i.e YOU CANT WRITE THEM AS MATRIX TIMES VECTOR.

ESSENTIALLY GRADIENT OF ~~VECTOR~~ FUNCTION AROUND A GIVEN POINT.

IF WE ASSUME EKF WILL ONLY MAKE SMALL CHANGES TO PREDICTIONS / ESTIMATES, WE COULD ASSUME A LINEAR SYSTEM AROUND A GIVEN POINT USING ∇F & ∇H AS F & H MATRICES.

$$F \rightarrow \nabla F\big|_{x(k|k)}$$

EVALUATED AT OUR LAST ESTIMATE $(k-1|k-1)$

$$H \rightarrow \nabla h\big|_{x(k|k-1)}$$

EVALUATED AT OUR BEST GUESS WHICH IS NOW $x(k|k-1)$ WE USED $f(x)$ to get $x(k|k-1)$ from $x(k-1|k-1)$

NOTE: ONLY USE THESE IN COVARIANCE CALCULATIONS

→ state & obs prediction USE MODELS THEMSELVES

$$x(k|k-1) = f(x(k-1|k-1), u(k), q(k))$$

$$z(k|k-1) = h(x(k|k-1), w(k))$$

3c) If YOU LINEARISE AROUND WRONG PLACE (I.E ESTIMATES ARE WRONG – ALL HELL BREAKS LOOSE – THE LINEARISATION DOES NOT EXPLAIN THE LOCAL TRAJECTORY OF THE PLANT/OBS FUNCTION AROUND THE TRUE STATE/TRUE MEASUREMENT → UNSTABLE

d) Update of P: N^2 elements in P where $x \in R^N$

$$P^+ = \underbrace{(I - WH)}_{N \times N} P^-$$

→ each element in P^+ involves N multiplications/additions

→ $O(N^3)$

Calculation of $S^{-1} = (HPH^T + R)$

$= O(m^3)$; $z \in R^M$

usually $\dim(x) > \dim(z)$
so P update dominates as $O(N^3)$

e) $(I - WH)P^-(I - WH)^T + WRW^T$

$P - WHP^- - PH^TW^T + \underbrace{WHP^-H^TW^T + WRW^T}_{WSW^T \quad (S = HP'H^T + R)}$

now $W = P^-H^TS^{-1} \rightarrow WSW^T = P^-H^TW^T$

$\rightarrow WS = P'H^T \rightarrow WSW^T = P'H^TW^T$

$SW^T = HP' \rightarrow WSW^T = WHP'$

so,

$P - \underbrace{WHP^-}_{WSW^T} - \underbrace{PH^TW^T}_{WSW^T} + WSW^T \Rightarrow P' - WSW^T$

4b) Joseph form is more stable

$\rightarrow$ no ~~scaled~~ subtraction of matrices

ABA^T is PSD SYMMETRIC FOR ALL A

PSD + PSD = PSD

5) 6) 7) SEE EXAMPLE CODE

IMPORTANT POINTS :

- 6) • IF ONLY 1 BEACON OBSERVED EKF STILL WORKS – OBSERVABILITY (INSTANTANEOUS) IS NOT A PROBLEM
  • CAN FLY THROUGH SINGULARITIES – SEE ABOVE

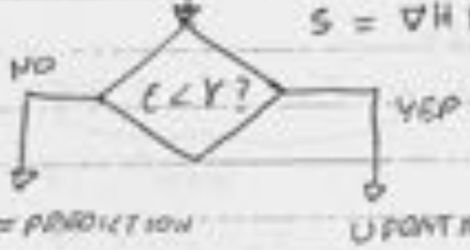
- 7) $\nu^T S^{-1} \nu$ is a SCALAR

  ACCEPT IF $\nu^T S^{-1} \nu < \gamma$ where

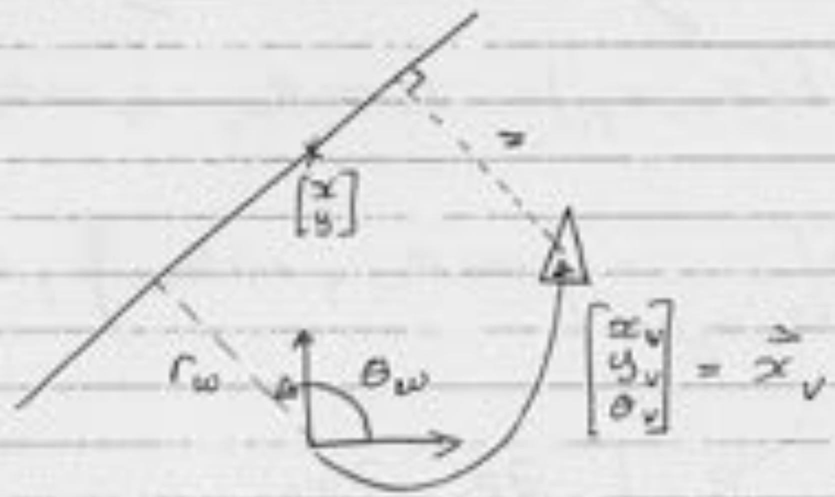
  $\gamma = \chi_1^2(95\%) = 5.024$

$z(k|k-1) = h(x(k|k-1))$

$S = \nabla H \, P(k|k-1) \nabla H^T + R$



8)



CONSIDER POINT ON LINE $\begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} x - r_w \cos\theta_w \\ y - r_w \sin\theta_w \end{bmatrix} \cdot \begin{bmatrix} r_w \cos\theta_w \\ r_w \sin\theta_w \end{bmatrix} = 0$$

$$\underbrace{x \cos\theta_w + y \sin\theta_w}\ = r_w$$

c.f. $ax + by + c = 0$ $\quad a = \cos\theta_w$, $b = \sin\theta_w$

distance from $ax + by + c = 0$ to $\begin{bmatrix} x_v \\ y_v \end{bmatrix}$
is $\frac{ax_v + by_v + c}{\sqrt{a^2+b^2}}$

$$x_v \cos\theta_w + y_v \sin\theta_w - r_w = \cancel{h(\vec{x})}$$
$$\equiv h(\vec{x}_v, x_{f_i})$$

THIS PREDICTS OBSERVATION

so

$$h(x_{v}, x_{f_i}) = x_v \cos\theta_{w_i} + y_v \sin\theta_{w_i} - r_{w_i}$$

$$\nabla h\Big|_{x_v, x_{f_i}} = \left[\cos\theta_{w_i}, \sin\theta_{w_i}, 0\right]$$

$$\begin{bmatrix} \frac{\partial z_1}{\partial x_1} & \frac{\partial z_2}{\partial x_2} & \cdots & \\ \frac{\partial z_2}{\partial x_1} & & & \\ \vdots & & & \\ \frac{\partial z_m}{\partial x_1} & \cdots & \cdots & \frac{\partial z_m}{\partial x_n} \end{bmatrix}$$

here z is $[1 \times 1]$ x is $[3 \times 1]$ so

∇h is $[1 \times 3]$