Mobile Robotics - Question Sheet 2

Note the matlab coding can be done simply by modifying the example files on the course web-site.

Kalman Filtering

1. Write down the Linear Kalman filter equations and then:
   - Annotate them
   - Explain why $\mathbf{P}$ and $\mathbf{x}$ are key variables
   - What is is the innovation? How can it be used to determine if the filter is well tuned.
   - Explain why jacobians appear the equations - about which point are they evaluated for $\mathbf{f}$ and $\mathbf{h}$ - why.
   - Explain the $i|j$ notation. Why is it not used in truth models? What would $\mathbf{x}(10|5)$ and $\mathbf{x}(5|10)$ mean?
2. A probe lands on the surface of Europa. It bores (melts) a hole through the ice and drops a simple "robot" into the ocean below. The vehicle can only control its buoyancy by controlling the volume of CO_2 in a "swim bladder". A requirement of the mission is that the vehicle descends at a constant velocity even though the density of the ocean may change. The vehicle is equipped with an upward-looking sonar which measures the time of flight from vehicle to ice-crust and back again.
   - Choose a suitable 2D-state for the system.
   - Write down suitable plant and sensor models (note these are truth models and so should not have $i|j$ terms)
   - What do the noises in these models represent.
   - Iterate the prediction and update stages by hand and show how the correlations between position and velocity are pivotal in allowing the KF to use position measurements to infer velocity.
   - Write down a model for a suitable (simple) controller to control velocity.
   - Draw a block diagram of the combined navigation and control system
3. Write down the Extended Kalman Filter (EKF) equations and then:
   - Annotate them

- Explain why jacobians appear the equations - about which point are they evaluated for $\mathbf{f}$ and $\mathbf{h}$? - why?
- What impact can this evaluation have on the stability of the filter.
- What is the computational complexity of and EKF as a function of state dimension and observation dimension.

4. Show that the following two forms of the Kalman Covariance update equations are equivalent:

$$\mathbf{P}(k|k) = (\mathbf{I} - \mathbf{WH})\mathbf{P}(k|k-1)(\mathbf{I} - \mathbf{WH})^T + \mathbf{WRW}^T$$
$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{WSW}^T$$

   Does one form offer any advantage over the other?

Mapping and Localisation

5. Write matlab code for 3D localisation of an autonomous underwater vehicle using n beacons at known locations. Use a non-linear Kalman filter. You can assume that the distance moved between transmission (to the beacons) and reception of acoustic signals (from them) is small. You will need to simulate a truth model and the observations which will be time of flight to and from the beacons.

6. Run the above code observing a random number of beacons at at time. Try the same experiment with the nonlinear least squares code from QSheet1. Explain the apparent success of the EKF over the batch method.

7. The term $\epsilon = \nu^T \mathbf{S}^{-1} \nu$ is distributed as a Chi-squared random variable with $dim(\mathbf{z})$ degrees of freedom. What are the dimensions of ϵ? Change your matlab code (measurement simulator) to occasionally return a grossly corrupted measurement. Run the simulation again this time plotting ϵ. From tables choose a suitable threshold on ϵ which if exceeded will cause the measurements to be rejected. Draw a block diagram of the logic required and implement it in your simulation.

8. An in-air sonar system on a paper-roll delivery AGV measures (noisely) the perpendicular distance to a wall from the sensor. The wall has previously been mapped and is parameterised as $[r_w, \theta_w]$ where r is the perpendicular distance to the world origin and θ_w is the angle of this perpendicular. For example the line $y = 2$ would parameterised as $[2, \pi/2]$. Assuming a 2D world model, write down an observation model describing observation of the $i^t h$ wall in a map of n for use in a SLAM algorithm. Evaluate the jacobian as a function of 2D-vehicle pose $\mathbf{x}_v$ and i^{th} Wall state $\mathbf{x}_{fi}$. Are there any disadvantages of this representation of a wall?