# Question Sheet 1

### Path Planning

1. What does holonomicity mean? What impact does it have on mobile robotics? Which of the following are holonomic

   - A helicopter
   - Space shuttle in orbit
   - Space Shuttle at the "Langragian Point" (finding out what the this is will give you a clue)

   Sketch a design for a hovering holonomic vehicle.

2. Figure  is a 2D "block world" in which a circular, differential-steer vehicle is operating. Assume the vehicle has perfect knowledge of the world and perfect navigation. It is required to drive the vehicle to the goal pose.

   - Draw the 2D configuration space of the vehicle in this world.
   - The visibility graph is the set of "clear line of sight" lines between vertices of configuration space obstacles. Draw this construction on your configuration space diagram. How may this construction be used to path plan. Can you see any disadvantages? Does it always yield a unique solution? If not suggest a criteria which can be used to select the best path.
   - Sketch the Voronoi diagram for the $\mathcal{C} - space$. Explain how this may be used for path planning. Contrast the behaviour it endows on the robot with the visibility graph approach. What are the disadvantages of the Voronoi approach.
   - Sketch the path taken by a robot employing the "bug algorithm". Can you design a block world in which the bug algorithm fails?

3. Explain how the "potential" method of path planning works. In what situations can it fail?

   - Write down an algebraic potential function (a function of vehicle position and obstacle properties) for a for a cylindrical obstacle
   - Write pseudo code for a potential function for a line-segment obstacle between two points $P_1$ and $P_2$.
   - Sketch the 3D shape of this function (potential in the vertical direction)
   - In matlab implement a potential path planner for a point vehicle moving in a world of point, circle and line objects. Print out screen shots of the trajectories taken for different obstacle arrangements.

**Estimation**

4. 
   - What is a recursive estimator?
   - Describe in the motivation behind Maximum Likelihood and Maximum-A Posteriori estimators.
   - How can a MAP estimator be turned into a recursive estimator.
   - What advantages do recursive estimators have over a batch estimators (all measurements processed at once)? On what grounds might a batch estimator be considered more robust to erroneous sensor data( hint: consider ways in which outliers could be detected)
   - Is non-linear least squares estimator a recursive estimator in the same sense as Kalman Filter?

5. Write matlab code for 3D localisation of an autonomous underwater vehicle using 3 beacons at known locations. Use a non-linear least squares algorithm. You can assume that the distance moved between transmission (to the beacons) and reception of acoustic signals (from them) is small. You will need to simulate a truth model and the observations which will be time of flight to and from the beacons .

6. The uncertainty in the least squares estimate of the vehicle location is given by

$$\mathbf{P} = \left\{ \sum_i \nabla \mathbf{H_i}^T \mathbf{R}^{-1} \nabla \mathbf{H_i} \right\}^{-1} \tag{1}$$

   where the $i^{th}$ observation is given by $\mathbf{z}_i = \mathbf{h}_i(\mathbf{x}_v) + \mathbf{w}$ and the covariance of the noise $\mathbf{w}$ is $\mathbf{R}$. Plot the x-y covariance as an ellipse centered on the x-y estimates of positions. How does the shape of the ellipse vary as a function of vehicle position (x,y and height(z)) and beacon geometry? For example, plot the uncertainty ellipses for three co-linear beacons and then with the beacons in a triangle. For each case let the vehicle range over a grid of positions . Produce a 3D plot with the z-axis proportional to the area enclosed in the 1-Sigma bound for the estimate at each x-y location at a constant height above the seabed. (hint : the area of an ellipse is proportional to the product of its eigenvalues. The determinant of a conic matrix (a covariance matrix) is the product of its eigne values). Comment on your results.

7. Show how the covariance produced by Equation 1 can be obtained without the summation. (hint: Concatenate the observations and their associated matrices)

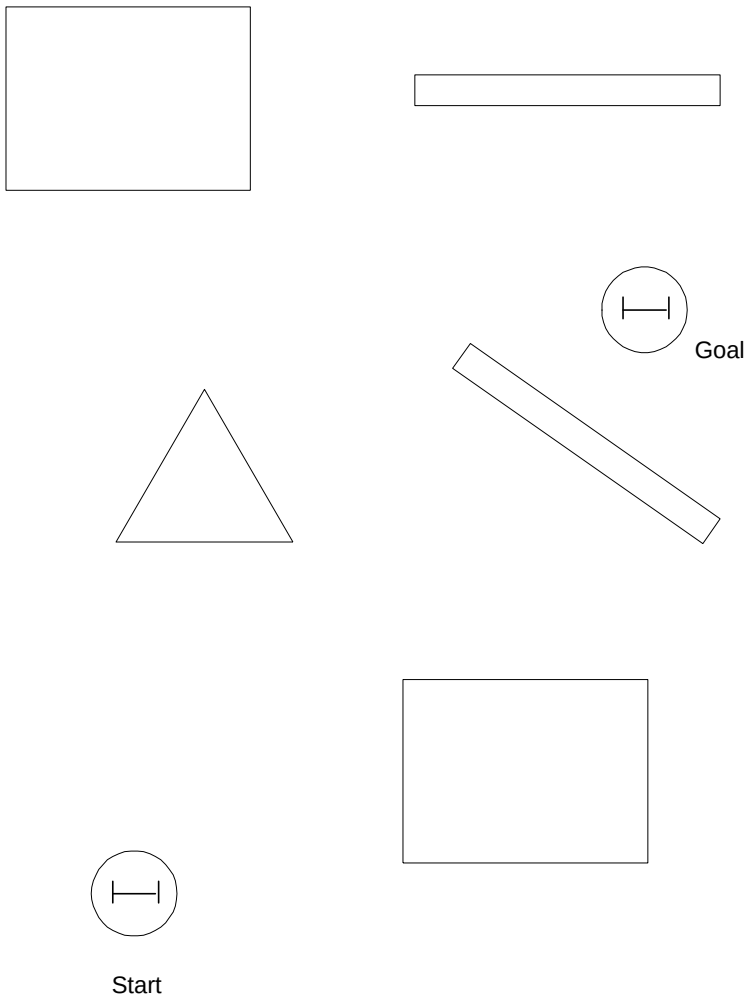


Figure 1: Block World for Question 2